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Mechanism of resonance and cancellation for train-induced vibrations on bridges with elastic bearings

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Abstract

In this paper, the mechanism involved in the phenomena of resonance and cancellation in the train-induced vibrations of railway bridges with elastic bearings is explained using an analytical approach. The train is modelled as a sequence of moving loads of constant intervals. The vibration shape of the elastically supported beam is approximated by the combination of a flexural sine mode and a rigid body mode. The present results indicate that under certain conditions, *resonances* of *much higher peaks* can be excited on elastically supported beams by moving trains at *much lower speeds* than those on simply supported beams. The *cancellation* is a phenomenon more decisive than that of resonance, in that it can suppress the latter even when the condition of resonance is met. Moreover, the speed for *cancellation* to occur is generally independent of the support stiffness. To verify the analytical results presented herein, a field test was conducted on two adjacent elastically supported bridges in existing railway lines. In the design of railway bridges, it is important that the phenomenon of resonance not be overlooked, as it is harmful not only to the riding comfort of passengers, but to the maintenance of railway tracks.

1. Introduction

Taiwan is located at one of the most active seismic belts in the Pacific Rim. In order to prevent the bridge structures from damages or collapse under severe earthquakes, various protective measures have been adopted by structural engineers. Elastic bearings represent a kind of devices commonly installed at the supports of the bridge girders for isolating the earthquake forces transmitted from the ground. While they are effective for isolating the ground-borne seismic

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forces, they can equally prevent the vehicle-induced vibrations from dissipation to the supports and then to the ground. This is certainly one disadvantage with the use of elastic bearings. For high speed railway bridges installed with elastic bearings, it is likely that the huge amount of vibration energy brought by the moving train be accumulated in the bridge, and that the bridge response be amplified when the train traverses at certain critical speeds. The repetitive occurrence of amplified vibrations may result in some early fatigue problems, affecting not only the riding quality of passengers, but also the service life of the track components.

In the past years, numerous researchers have studied the moving load and vehicle–bridge interaction problems, including Frỳba [1], Kurihara and Shimogo [2], Yang and co-workers [3–5], Yau et al. [6], Cheung et al. [7], and Au et al. [8], among others. Due to the regular, repetitive nature of the wheel loads constituting a train, both the phenomena of resonance and cancellation may be induced on the bridge by the train moving at high speeds. The *resonance* phenomenon relates to the continuous *build-up* of the free-vibration response on the bridge as there are more loads passing by. In contrast, the *cancellation* phenomenon implies that the waves associated with the free-vibration responses of the bridge generated by the sequential moving loads cancel out each other. If the resonance condition can be reached by a train within its operation speeds, then some detrimental effects can be expected on the track system, as well as on the moving train itself. Such a problem will be aggravated when the factor of elastic bearings is taken into account.

Previously, rather few research works have been conducted on the dynamic response of elastically supported beams to moving loads. In the analytical study by Yau et al. [9], envelope formulas were developed for elastically supported beams with light damping subjected to moving loads. Recently, Lin [10] investigated the vibrations of railway bridges installed with elastic bearings, together with measures for vibration reduction. In this study, focus is placed on the *physical interpretation* of the mechanism involved in the phenomena of resonance and cancellation of the dynamic response of elastically supported beams to moving loads. The key factors affecting the dynamic response of bridges will be investigated, with comments made concerning the suitability of using elastic bearings as aseismic devices for railway bridges. Also presented are the results obtained from a field test that serve to verify the theory presented in this paper.

2. Formulation of the theory

The bridge model adopted is the one shown in Fig. 1, in which a beam supported by two identical elastic bearings is considered. The following assumptions will be adopted in the derivation of a closed-form solution for the elastically supported beam under the moving loads [9]: (1) The vertical stiffness of each elastic bearing is K. The mass of the spring is negligible compared with that of the bridge. (2) The train is modelled as a sequence of equally spaced moving loads, with the inertial effect neglected. (3) For beams subjected to moving loads, which is basically a transient problem with very short acting time, only the first mode of vibration of the bridge need be considered, while the higher modes can be neglected without losing much accuracy [11]. (4) The damping of the beam can be neglected, also due to the transient nature of the moving load problem.

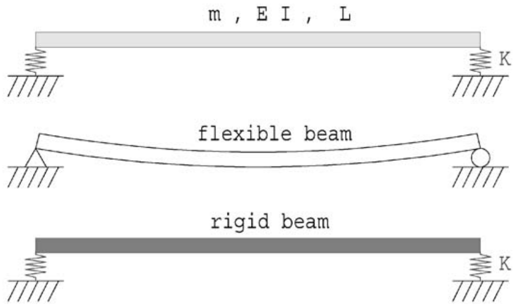

Fig. 1. Model beam as a superposition of simple and rigid beams.

2.1. Assumed modal shape of vibration

By the concept of modal superposition, the deflection $u(x, t)$ of the elastically supported beam can be expressed as

$$u(x,t) = \sum \phi_n(x) q_n(t), \tag{1}$$

where $q_n(t)$ denotes the generalized coordinate and $\phi_n(x)$ the shape function of the nth vibration mode. As was stated above, only the first mode of vibration will be considered in analyzing the vibrational response of the beam, as it is essentially a transient problem with very short acting time [4,10]. If a mathematically exact approach is employed to find the first modal shape of vibration for the elastically supported beam, the final form of the modal shape will be rather complex, rendering it impossible to obtain simple closed-form solutions. As the first priority herein is to derive a closed-form solution, by which the mechanism behind the key phenomena can be interpreted, the vibration shape of the elastically supported beam will be approximated as the *superposition* of the first modal shape of the *flexural deflection* of the beam *with simple supports* and the first modal shape of a *rigid beam supported by the elastic springs*, as indicated in Fig. 1 [9], that is,

$$\phi(x) = \sin\frac{\pi x}{L} + \kappa, \tag{2}$$

where $\kappa = (EI\pi^3)/(KL^3)$ denotes the ratio of the flexural rigidity EI of the beam to the stiffness K of the elastic bearing, and L the length of the beam. As can be seen, a higher stiffness ratio κ means a softer elastic spring and a zero stiffness ratio means the special case of simple supports. The shape function $\phi(x)$ in Eq. (2) differs from that used in Ref. [9] by a factor $1/(1+\kappa)$, which is acceptable, since shape functions do not have absolute magnitudes.

2.2. Elastically supported beam subjected to a single moving load

For an elastically supported beam subjected to a single moving load P of speed v, the equation of motion for the deflection $u(x, t)$ of the beam is

$$m\ddot{u} + EIu'''' = P\delta(x - vt) \quad \text{for } 0 \leqslant vt \leqslant L, \tag{3}$$

where m is the mass per unit length and EI the flexural rigidity of the beam. The boundary conditions are

$$\begin{gathered} EIu''(0,t) = 0, \quad EIu''(L,t) = 0, \\ EIu'''(0,t) = -Ku(0,t), \quad EIu'''(L,t) = Ku(L,t). \end{gathered} \tag{4}$$

By multiplying both sides of Eq. (3) by the the shape function $\phi(x)$ in Eq. (2) and integrating with respect to the length L of the beam, one obtains

$$\ddot{q} + \omega^2 q = \frac{2P}{mL}\left(1 + \frac{8\kappa}{\pi} + 2\kappa^2\right)^{-1}\left(\sin\frac{\pi vt}{L} + \kappa\right), \tag{5}$$

where the frequency of vibration ω is

$$\omega = \omega_0\sqrt{\frac{\pi + 4\kappa}{\pi + 8\kappa + 2\pi\kappa^2}}. \tag{6}$$

Here, ω_0 indicates the frequency of vibration of the associated beam with simple supports,

$$\omega_0 = \left(\frac{\pi}{L}\right)^2\sqrt{\frac{EI}{m}}. \tag{7}$$

It has been demonstrated that the fundamental frequency ω of vibration solved using the present approximate shape function $\phi(x)$, as given in Eq. (6), appears to be in excellent agreement with the exact one [9].

The generalized coordinate $q(t)$ can be solved from Eq. (5), together with zero initial conditions, as

$$q(t) = \frac{2PL^3}{EI\pi^4}\left(1 + \frac{4\kappa}{\pi}\right)^{-1}\left[\left(\frac{\sin\Omega t - S\sin\omega t}{1 - S^2}\right) + \kappa(1 - \cos\omega t)\right], \tag{8}$$

where Ω denotes the driving frequency implied by the moving load, and S is a *speed parameter* defined as the ratio of the driving frequency Ω to the bridge frequency ω,

$$\Omega = \frac{\pi v}{L}, \qquad S = \frac{\Omega}{\omega} = \frac{\pi v}{\omega L}, \tag{9}$$

In Eq. (8), the term containing the stiffness ratio κ represents the effect of the elastic supports. For the special case of simple supports, i.e., with $\kappa = 0$, the present solution reduces to that given in Ref. [4] for simply supported beams.

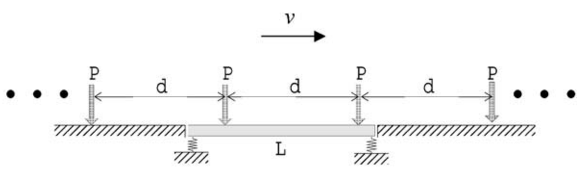

Fig. 2. Elastically supported beam subjected to equidistant moving loads.

2.3. Elastically supported beam subjected to a series of moving load

As shown in Fig. 2, consider now that the elastically supported beam is subjected to a series of concentrated loads P of equal intervals d moving at speed v, to simulate the loading action of a train that consists of N cars of length d. The equation of motion for the beam should now be modified as

$$m\ddot{u} + EIu'''' = P\sum_{k=1}^{N} \delta[x - v(t - t_k)] \cdot [H(t - t_k) - H(t - t_k - \Delta t)], \tag{10}$$

where δ denotes the Dirac delta function, H the unit step function, $t_k = (k-1)d/v$ the arriving time of the kth load at the beam, $\Delta t = L/v$, and N is the total number of moving loads. The term $H(t - t_k)$ indicates the arrival of the kth load at the beam and the term $H(t - t_k - \Delta t)$ the departure from the beam. The boundary conditions given in Eq. (4) remain valid.

Based on the hypothesis of linear theories, the deflection of the beam induced by a sequence of moving loads can be obtained as the superposition of the deflection of the beam induced by each of the moving loads, if due account is taken of the time lag of each moving load. Consequently, the generalized deflection $q(t)$ of the beam for the present case can be obtained as a generalization of Eq. (8) as

$$q(t) = \frac{2PL^3}{EI\pi^4}\left(1 + \frac{4\kappa}{\pi}\right)^{-1}[Q_1(t) + Q_2(t)], \tag{11}$$

where $Q_1(t)$ represents the contribution caused by the *flexural vibration* of the beam with simple supports, and $Q_2(t)$ the *rigid displacement* of the elastic bearings, namely,

$$\begin{aligned} Q_1(t) = \frac{1}{1 - S^2}\sum_{k=1}^{N} \{&[\sin \Omega(t - t_k) - S \sin \omega(t - t_k)]H(t - t_k) \\ &+ [\sin \Omega(t - t_k - \Delta t) - S \sin \omega(t - t_k - \Delta t)]H(t - t_k - \Delta t)\}, \end{aligned} \tag{12a}$$

$$\begin{aligned} Q_2(t) = \kappa \sum_{k=1}^{N} \{&[1 - \cos \omega(t - t_k)]H(t - t_k) \\ &- [1 - \cos \omega(t - t_k - \Delta t)]H(t - t_k - \Delta t)\}, \end{aligned} \tag{12b}$$

where Ω denotes the driving frequency implied by the moving loads and ω the frequency of vibration of the elastically supported beam. For the special case of $\kappa = 0$, the solution given in Eq. (11) reduces to that given in Ref. [4] for a beam with simple supports.

3. Conditions of resonance and cancellation

The generalized deflection of the elastically supported beam given in Eq. (11) consists of two parts, that is, the *forced vibration* caused by the moving loads when they are directly acting on the beam, as indicated by the terms containing the driving frequency Ω, and the *residual free vibration* caused by the moving loads that have passed the beam, as indicated by the terms containing the bridge frequency ω. When all the moving loads have passed the beam, the forced vibration part terminates immediately. However, the free vibration part, which is of sinusoidal form, continues to exist until it is eventually damped out.

Both the phenomena of resonance and cancellation relate to the free vibrations induced by the moving loads. When a moving load has passed the beam, waves of the sinusoidal form will be induced on the beam. If the time lag of the wave components induced by each moving load equals a multiple of the period $2\pi/\omega$, then superposition of all the wave components will result in amplified responses. This is the so-called phenomenon of *resonance*. On the contrary, if the time lag equals an odd multiple of half of the period, the wave components induced by the sequentially moving loads will just cancel out, indicating that the phenomenon of *cancellation* has occurred.

Whether the phenomena of resonance or cancellation will occur or not depends only on the free vibration part of the motion. In order to interpret the two phenomena using the analytical solution, let us consider the *critical* case when the $(N-1)$st moving load has left the beam and the Nth load has entered the beam, that is, when $t_N < t \leqslant t_N + \Delta t$. Such a case is considered critical, since the beam is excited to the utmost. From Eq. (11), one can obtain the following for such a case:

$$q(t) = \frac{2PL^3}{EI\pi^4}\left(1 + \frac{4\kappa}{\pi}\right)^{-1}[A(t)H(t - t_N) + B(t)H(t - t_N - \Delta t)], \tag{13}$$

where the dynamic response factors $A(t)$ and $B(t)$ are

$$A(t) = \frac{1}{1 - S^2}[\sin\Omega(t - t_N) - S\sin\omega(t - t_N)] + \kappa[1 - \cos\omega(t - t_N)], \tag{14a}$$

$$\begin{aligned} B(t) = {} & \left(\frac{-2S}{1 - S^2}\cos\frac{\omega L}{2v} + 2\kappa\sin\frac{\omega L}{2v}\right) \\ & \times \left[\sin\omega\left(t - \frac{L}{2v}\right) + \frac{\sin\omega(t - (L/2v) - (t_N/2))\sin\omega((t_N/2) - (d/2v))}{\sin(\omega d/2v)}\right]. \end{aligned} \tag{14b}$$

In Eq. (13), the term $A(t)H(t - t_N)$ indicates the forced vibration of the beam caused by the Nth moving load, and the term $B(t)H(t - t_N - \Delta t)$ the sum of all the free vibrations caused by the previous $N-1$ moving loads that have already passed the beam.

Some physical interpretations can be given using Eq. (14b). First of all, if the denominator within the brackets vanishes, i.e., when $\sin(\omega d/2v) = 0$, the response of the beam reaches a peak. By L'Hospital's rule, the second term within the brackets of the dynamic response factor $B(t)$ under the resonance condition becomes

$$\frac{\sin\omega\big(t - (L/2v) - (t_N/2)\big)\sin\omega((t_N/2) - (d/2v))}{\sin(\omega d/2v)} = (N - 2)\sin\omega\left(t - \frac{L}{2v}\right). \tag{15}$$

Accordingly, the dynamic response factor $B(t)$ can be written as

$$B(t) = \left(\frac{-S}{1 - S^2}\cos\frac{\omega L}{2v} + \kappa\sin\frac{\omega L}{2v}\right) \times 2(N - 1)\sin\omega\left(t - \frac{L}{2v}\right). \tag{16}$$

As can be seen, *larger* amplitude for the response can be expected as there are *more* loads passing the beam, indicated by the term $2(N-1)$. Such a phenomenon is similar to that observed for beams with simple supports [4]. Corresponding to $\sin(\omega d/2v) = 0$, the condition of resonance is $\omega d/2v = i\pi$ with $i = 1, 2, 3, \ldots$, or $v = \omega d/(2i\pi)$, which can also be written in terms of the *speed* *parameter S* based on the definition of Eq. (9) as

$$S_r = \frac{\pi v}{\omega L} = \frac{1}{2i}\frac{d}{L} \quad \text{with} \quad i = 1, 2, 3, \ldots \tag{17}$$

which is the same as that for simply supported beams. As can be seen either from Eq. (17) or Fig. 3, the resonant speed parameter S_r is a function of the ratio d/L of the car length to the bridge length. For trains of the commercially available models of which the car length d is known, the resonant speed parameter S_r, which is dimensionless, is generally small for bridges of practical length L. An observation from Eq. (17) is that the *longer* the span length L of a beam, the *easier* is for the resonance phenomenon to occur. It is true that the resonant speed parameter S_r computed from Eq. (17) is the same for both the elastically supported and simply supported beams. However, because the fundamental frequency of the former is much lower than that of the latter, the resonant speed, i.e., $v_r = \omega L S_r/\pi$, for vehicles moving over an elastically supported beam will be *much lower* than that over a simply supported beam. Besides, it should be noted that according to Eq. (13), the amplitude of the resonance response also depends on the stiffness ratio κ of the elastic supports.

On the other hand, by setting the parenthesized term in Eq. (14b) equal to zero, all the residual free vibrations caused by the previous $N-1$ moving loads just cancel out. This is exactly the condition of *cancellation* for the elastically supported beam:

$$Y(S) = \frac{-S}{1 - S^2}\cos\frac{\omega L}{2v} + \kappa \sin\frac{\omega L}{2v} = 0. \tag{18}$$

For simply supported beams, $\kappa = 0$, the preceding condition reduces to $\cos(\omega L/2v) = 0$, or $\omega L/2v = \pi(2i-1)/2$ with $i = 1, 2, 3, \ldots$, from which the speed parameter S can be determined as

$$S_c = \frac{1}{2i-1} \quad \text{with} \quad i = 1, 2, 3, \ldots \tag{19}$$

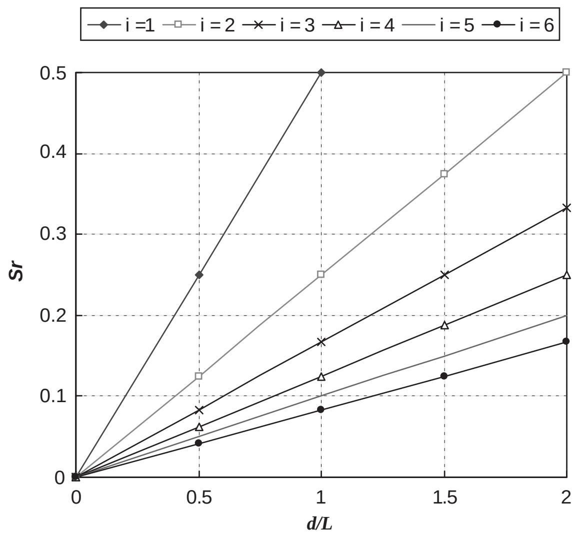

Fig. 3. $S_r - d/L$ plot for resonance.

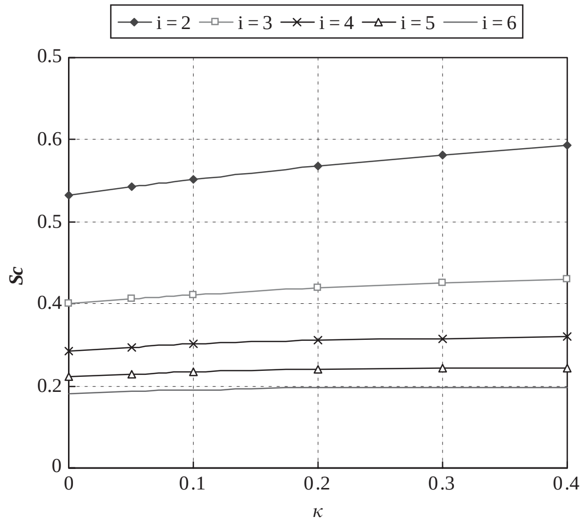

Fig. 4. $S_c-\kappa$ plot for cancellation.

which is valid only for the simply supported beam, i.e., for the case with $\kappa = 0$. The cancellation speed for the elastically supported beam cannot be presented explicitly, since Eq. (18) is an implicit function. The solution computed from $Y(S) = 0$ in Eq. (18) for the cancellation speed parameter S_c has been plotted with respect to the stiffness ratio κ in Fig. 4. As can be seen, the cancellation speed parameter S_c increases slightly as the stiffness ratio κ increases. For example, for the case with $i = 2$, we have $S_c/S_{c(\kappa=0)} \approx 1 + 0.5\,\kappa$. However, since the fundamental frequency ω of the elastically supported beam also decreases as the stiffness ratio κ increases, i.e., $\omega/\omega_0 \approx 1 - 0.5\,\kappa$. It turns out that the difference between the real cancellation speed v_c, computed as $v_c = S_c\omega L/\pi$, for vehicles moving over an elastically supported beam and that over a beam with simple supports is generally small.

4. Mechanism of resonance and cancellation

In design practice, the impact factor I is used to account for the dynamic amplification effect on the bridge due to the passage of moving vehicles through increase of the design forces and stresses. The impact factor I used in this study is defined as follows:

$$I(x) = \frac{R_d(x) - R_s(x)}{R_s(x)}, \tag{20}$$

where $R_d(x)$ and $R_s(x)$ denote the maximum dynamic and static response, respectively, of the bridge calculated at poison x due to the moving loads.

In order to unveil the mechanism underlying the phenomena of resonance and cancellation of bridge responses in relation to the effect of elastic supports, two bridges, B1 and B2, will be considered, of which the key properties have been listed in Table 1. The train is simulated as 8 moving loads of equal weight $P = 220$ kN spaced at an interval $d = 25$ m. By changing the vertical stiffness of the elastic bearings, say, allowing it to vary in terms of the stiffness ratio κ from 0 to

Table 1
Properties of the bridges used in analysis

	L (m)	M (t/m)	EI (kN m^2)	ω_0 (rad/s)
B1 Bridge	23	30	1.4×10^8	40.3
B2 Bridge	27	32	2.0×10^8	33.9

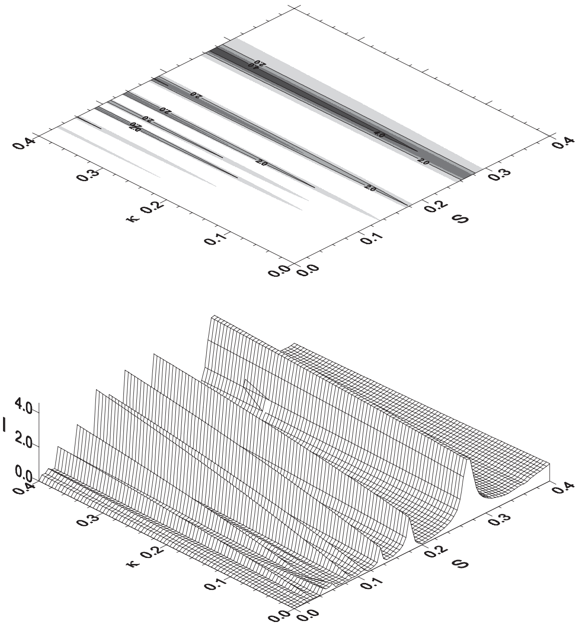

Fig. 5. $I-\kappa-S$ plot and contour map (Bridge B1).

0.4, and computing the impact factor I for the midpoint deflection of the beam due to the loads moving at different speeds, one can establish the $I-\kappa$—S plots as in Figs. 5 and 6 for the two bridges.

As can be seen from Fig. 5, for bridge B1, higher stiffness ratios κ generally result in higher resonant peaks, indicating that the elastic bearings inserted at the bridge supports tend to amplify the bridge response. Such a phenomenon can be clearly explained using Fig. 7, where two impact curves were plotted each for $\kappa = 0$ (i.e., for the beam with simple supports) and $\kappa = 0.2$. As can be seen from Fig. 7(a), the resonance phenomenon appears to be *not so visible* for the simply supported beam for $S<0.1$, which can be practically ignored. However, it is amplified drastically and becomes *rather significant* and *non-negligible* due to installation of the elastic bearings on the bridge. In the design of high speed railway bridges, the detrimental effect of elastic bearings in amplifying the low-speed resonant responses should be seriously taken into consideration.

On the other hand, a comparison of Fig. 7(a) with (b) reveals two interesting facts. First, the real resonant speed v_r for an elastically supported bridge is actually *much smaller* than that for the bridge with simple supports (see Fig. 7(b)), although the resonant speed parameter S_r remains

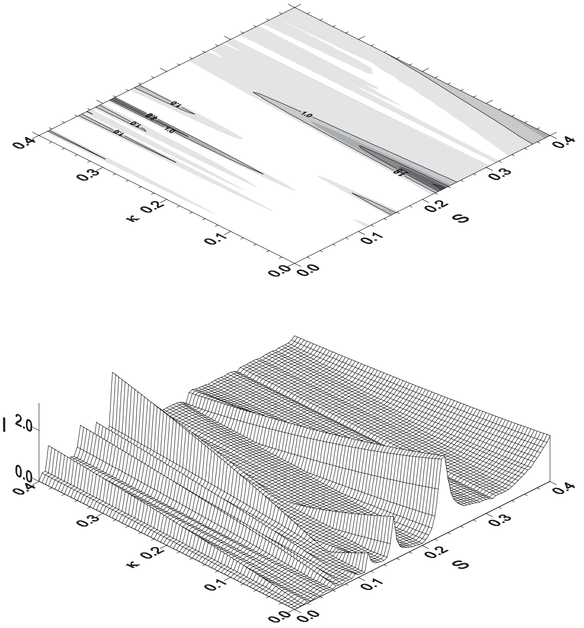

Fig. 6. $I-\kappa-S$ plot and contour map (Bridge B2).

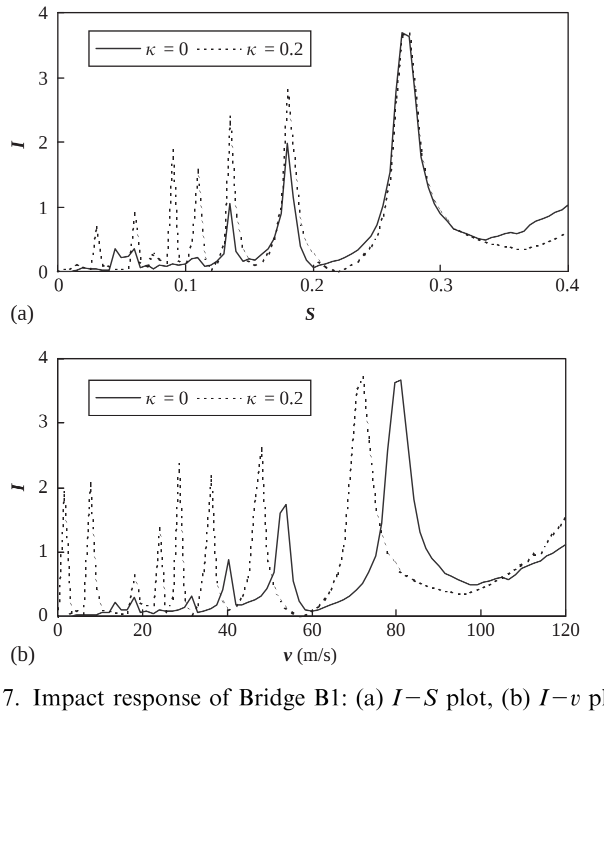

Fig. 7. Impact response of Bridge B1: (a) $I-S$ plot, (b) $I-v$ plot.

the same for both (see Fig. 7(a)). Second, the real cancellation speed v_c seems to be *close* for both the elastically and simply supported bridges (see Fig. 7(b)), although the corresponding speed parameter S_c is slightly larger for the elastically supported bridge (see Fig. 7(a)). Such findings are consistent with the statements made following Eq. (19). In studying the impact response on bridges, the results are often presented as an $I-S$ plot for its elegance in expression and convenience in extension to more general cases, as both I and S are non-dimensional. However, we should not misinterpret the real physical meanings implied by these non-dimensional parameters.

As can be seen from Fig. 6, for the speed parameter in the range $S<0.11$, larger resonant peaks can be expected for bridge B2 for larger stiffness ratio κ. However, the same is not true for the speed parameter in the range $S>0.11$. The reason can be given as follows. First, for bridge B2, the car/bridge length ratio is $d/L = 25/27 = 0.926$. By drawing a vertical line at $d/L = 0.926$, one can obtain from the resonance plot (i.e., Fig. 3) several intersections of which the ordinates (for S_r) represent the points of resonance, as indicated in Fig. 8(a). Because the resonance condition in terms of speed parameter is independent of the stiffness ratio κ, one can draw a *resonance plot* as shown in Fig. 8(b) for bridge B2, in which each horizontal line represents one of the ordinates for resonance (i.e., passes through one of the intersections) shown in Fig. 8(a). Finally, we can superimpose the resonance plot of Fig. 8(b) with the *cancellation plot* of Fig. 4 to obtain the *resonance/cancellation plot* as shown in Fig. 8(c), which contains all the information we need for explaining the dynamic response of bridge B2.

Take the resonance line $S = 0.23$ in the resonance/cancellation plot, i.e., Fig. 8(c), for example. The cancellation line below becomes *closer* to this line as the stiffness ratio κ increases, which means that the *resonance peaks will be suppressed* as the stiffness ratio κ increases. Here, it should be added that cancellation is a condition *more decisive* than resonance, since the dynamic response factor $B(t)$ remains equal to zero once the condition of cancellation is met, regardless of the presence of resonance, as can be observed from Eq. (14b). For the same reason, we can explain why the resonant peak at $S = 0.11$ first diminishes and then grows as the stiffness ratio κ increases. This is due to the fact that the resonance and cancellation lines are close in the beginning, but are getting apart for increasing κ. After realizing the mechanism of cancellation

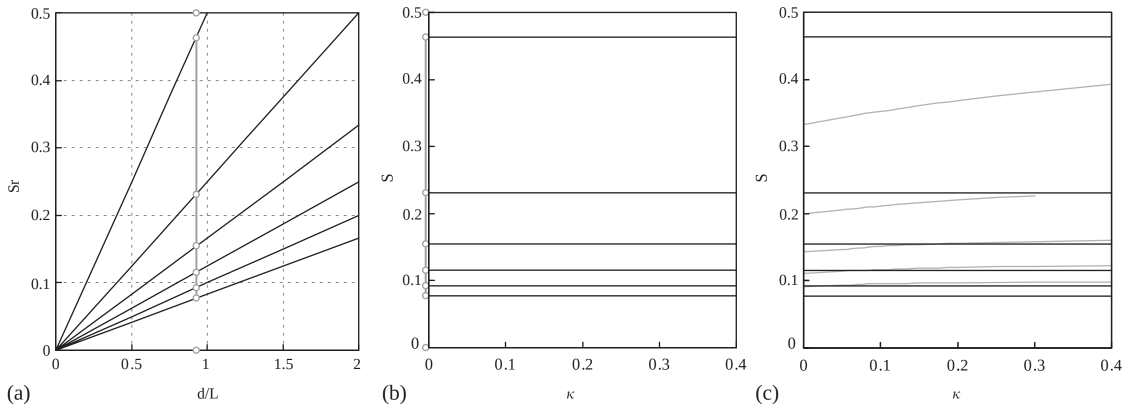

Fig. 8. Mechanism of resonance and cancellation: (a) points of resonance, (b) resonance plot, (c) resonance/cancellation plot.

and resonance for elastically supported bridges, we shall proceed to verify the present theory by some field measurements.

5. Field measurement of railway bridges

There have been reports by train drivers on the unusually high levels of oscillations when maneuvering the trains to traverse the bridges over the Fongshan Creek on the Western Railway Lines in northern Taiwan. In order to clarify the reasons behind the problem, a field measurement was carried out for two adjacent bridges, A1 and A2, located at the Fongshan Creek, which were not known to be elastically supported at the time of testing. Two locomotives of type E300 (see Fig. 9) linked back-to-back together were used to generate the action of moving loads at different speeds. Some typical dimensions of the bridges and locomotives were shown in Fig. 10. Each locomotive has a bogie-to-bogie distance of 9.6 m and a gross weight of 96 t, much heavier than the normal passenger cars used. The railway gauge is 1067 mm, which is typical in Taiwan. The fundamental frequencies of the two bridges measured from an ambient vibration test are: $f = 5.17$ Hz for bridge A1 and $f = 5.13$ Hz for bridge A2, which represent the combined dynamic effect of all the components constituting the railway bridge, including the continuous rails, sleepers, ballast, elastic bearings, and the girder and two side flanges that form a cross-section of the U shape. By design, the two bridges are identically the same, but due to degradation in material properties they turn out to be slightly different in the material properties.

During the testing, the two locomotives connected back-to-back are allowed to travel on one side, i.e., the test side, of the two-track railway lines including the bridge sections at the following speeds: 15, 30, 45, 60, 75, 85, and 110 km/h. The maximum accelerations measured at the

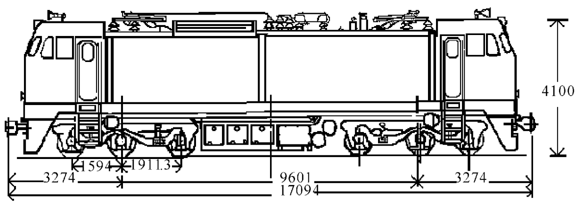

Fig. 9. E300 Locomotive (unit: mm).

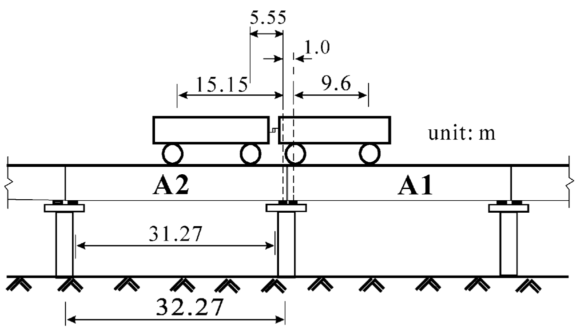

Fig. 10. Schematic of tested bridges.

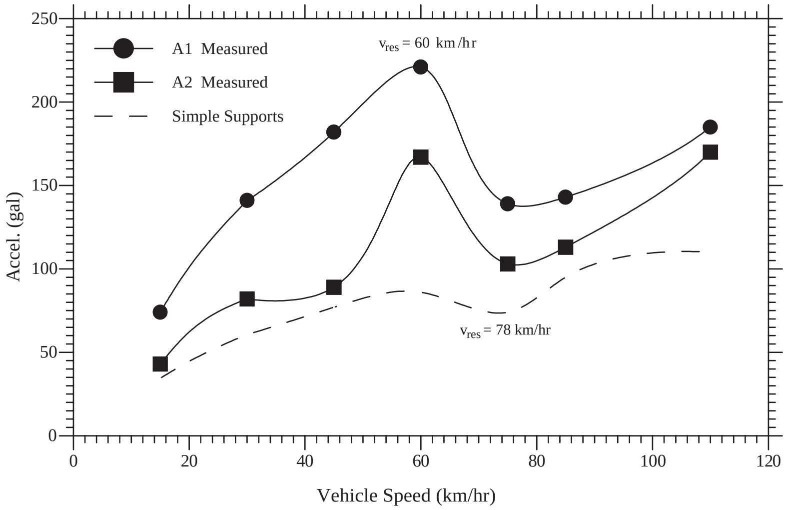

Fig. 11. Maximum midpoint accelerations.

midpoint of the A1 and A2 bridges using seismometers during the passage of the two locomotives at different speeds have been plotted in Fig. 11. Also plotted in Fig. 11 are the maximum accelerations *computed* for the midpoint of the two bridges by simulating the two locomotives as four equal-weight moving loads, but with *simple support* conditions, using the computer program developed by the research group for vehicle–bridge interactions at the National Taiwan University.

One observation from the measured and computed results shown in Fig. 11 is that they both show the occurrence of a peak response at the speed around 60 km/h. Such a speed is exactly one of the resonance speeds, which can be verified using Eq. (17), that is,

$$\text{Bridge A1}: \quad v_r = \frac{\omega d}{2n\pi} = \frac{fd}{n}\bigg|_{\substack{f\,=\,5.17\\ d\,=\,9.6\\ n\,=\,3}} = 16.55\,\text{m/s} = 59.6\,\text{km/h},$$

$$\text{Bridge A2}: \quad v_r = \frac{\omega d}{2n\pi} = \frac{fd}{n}\bigg|_{\substack{f\,=\,5.13\\ d\,=\,9.6\\ n\,=\,3}} = 16.42\,\text{m/s} = 59.1\,\text{km/h}.$$

However, the computed response in Fig. 11 appears to be much smaller than the measured ones for the two bridges, while the peak response around the speed of 60 km/h is not quite visible. This is primarily due to the adoption of simple supports for the bridges in the finite element analysis. We assumed the bridges to be simply supported because we were not informed of the existence of elastic bearings between the bridge girders and column tops. Nevertheless, the relatively high amplitudes of the measured responses for both bridges, compared with the computed one, did reveal the interesting fact that significant *magnification* may be introduced by existence of elastic bearings, which was latter known to us. The magnification effect of elastic bearings seems to have received little attention from researchers working on railway bridges in the past.

Based on the static deflection tests under the locomotive loads, which were conducted as part of the preliminary tests, the spring constants measured for the A1 and A2 bridges are $K_1 = 3.5 \times 10^6$ kN/m and $K_2 = 8 \times 10^6$ kN/m, respectively, assuming that the elastic bearings installed at the two ends of a bridge are the same. All the key properties identified for the tested bridges, including the flexural rigidity EI and stiffness ratio κ, are listed in Table 2. With the data given in Table 2, the midpoint responses computed by the finite element program for the A1 and A2 bridges using the moving loads assumption were plotted in Fig. 12. As can be seen, because of the inclusion of elastic bearings, the computed responses agree generally well with the measured ones for the two bridges. Moreover, larger response exists for bridge A1 simply because it has softer support bearings.

Let us now turn to the phenomenon of cancellation. From Eq. (19), for the case with simple supports, the speed parameter S for cancellation to occur is

$$S_c = \left.\frac{1}{2i-1}\right|_{i=8} = 0.067.$$

Table 2
Properties of tested bridges

	L (m)	f (Hz)	K (kN m)	EI (kN m^2)	$\kappa = (EI\pi^3)/(KL^3)$
A1 Bridge	31.3	5.17	3.5×10^6	2.40×10^8	0.069
A2 Bridge	31.3	5.13	8×10^6	2.44×10^8	0.031

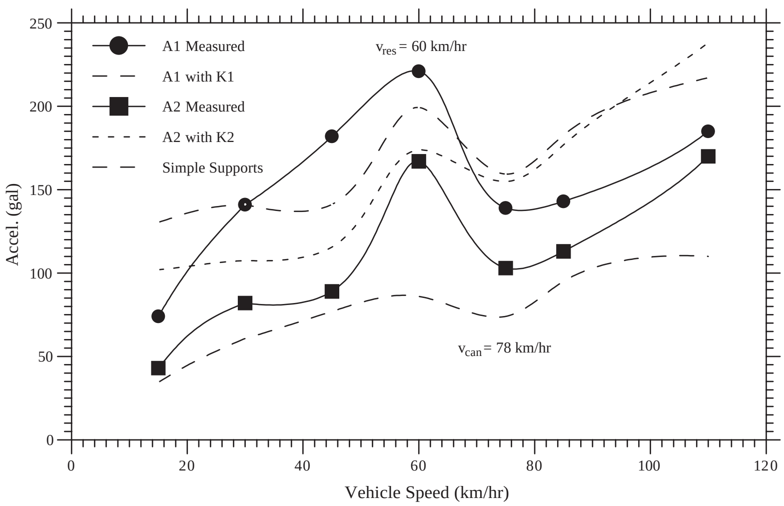

Fig. 12. Computed solutions for elastic supports.

Correspondingly, the cancellation speeds for the two bridges are

$$\text{Bridge A1}: \quad v_c = \frac{\omega_0 L S_c}{\pi} = 2fLS_c$$
$$= 2 \times 5.17 \times 31.3 \times 0.067 = 21.68\,\text{m/s} = 78\,\text{km/h},$$

$$\text{Bridge A2}: \quad v_c = \frac{\omega_0 L S_c}{\pi} = 2fLS_c = 2 \times 5.13 \times 31.3 \times 0.067$$
$$= 21.51\,\text{m/s} = 77.5\,\text{km/h} \simeq 78\,\text{km/h}.$$

This is exactly the cancellation speed for the bridge with simple supports, as can be seen from the numerical solution shown in Fig. 11.

To consider the effect of elastic bearings, the speed parameter S for the cancellation to occur can be solved from Eq. (18) or

$$Y(S) = \frac{-S_c}{1 - S_c^2} \cos\frac{\pi}{2S_c} + \kappa \sin\frac{\pi}{2S_c} = 0. \tag{21}$$

By substituting the stiffness ratios κ of 0.063, 0.031, as given in Table 2, into the preceding equation, the speed parameter S solved for the A1 and A2 bridges respectively are 0.069 and 0.068. Correspondingly, the cancellation speeds v for the two bridges as

$$\text{Bridge A1}: \quad v_c = 0.069 \times 2fL = 0.069 \times 2 \times 5.17 \times 31.3 = 22.3\,\text{m/s} = 80.4\,\text{km/h},$$

$$\text{Bridge A2}: \quad v_c = 0.068 \times 2fL = 0.068 \times 2 \times 5.13 \times 31.3 = 21.8\,\text{m/s} = 78.5\,\text{km/h}.$$

Clearly, the above (computed) speeds are consistent with the (measured) speeds for the occurrence of minimal responses for the two bridges, as shown in Fig. 12, which is an indication of the reliability of the present theory.

6. Concluding remarks

In this paper, the mechanisms underlying the resonance and cancellation phenomena of elastically supported bridges caused by a sequence of equidistant moving loads have been analytically studied. A field measurement on two adjacent bridges travelled by two back-to-back connected locomotives was also conducted to confirm the phenomena of resonance and cancellation identified. The conclusions drawn from this study are: (1) The resonance condition in terms of the speed parameter S is the same for the beam with both the elastic and simple supports. Since an elastically supported beam has a lower frequency of vibration, it therefore has a lower resonant speed v, meaning that it can be more easily excited than a beam with simple supports. (2) The speed parameter for the cancellation condition to occur increases slightly as the stiffness ratio increases. However, since the frequency of vibration is slightly smaller for an elastically supported beam, it turns out that the real cancellation speed for an elastically supported beam remains close to that for the simply supported beam. (3) Whenever the cancellation speed comes close to or coincides with the resonance speed, the phenomenon of resonance will be suppressed, meaning that the cancellation condition is more decisive than the resonance condition. (4) Once a resonance condition is reached for an elastically supported beam, much larger peak responses will be induced on the beam, compared with those of the simply supported beam.

There is no doubt that elastic bearings are effective devices for isolating the earthquake forces transmitted from the ground to the superstructure. However, the installation of these devices can also prevent the transmission or dissipation of vehicle-induced forces from the superstructure to the ground. Thus, the huge amount of vibration energy brought by a train may be accumulated and amplified on the bridge during its passage. Such a fact should not be overlooked in the design of railway bridges, especially those to be travelled by high-speed trains, since it is harmful not only for the riding comfort of passing trains, but also for the maintenance of track structures, as the repetitive occurrence of high-amplitude resonant peaks may cause fatigue problems on related components.

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